CHAPTER 16

FIBRE OPTICS AND WAVEGUIDES

16.1 INTRODUCTION :

In recent times transparent dielectric fibres have been developed to carry efficiently optical signals from one place to another. These are known as *optical fibres*. The study of the properties of such optical fibres is known as *fibre optics*. Usually the diameter of these fibres is large compared to the small wavelength of optical signals. So the inherent wave nature of light may be neglected and we can apply the known laws of geometrical optics to study the properties of optical fibres. The importance of fibre optics arises from the fact that an optical fibre can carry a huge amount of information by using a light beam as a carrier wave. Moreover, because of low cost, high reliability and extremely low losses optical fibres are gradually being used more and more.

If the diameter of the fibre becomes of the order of wavelength, the wave nature of light cannot be neglected (e.g., in fibres of thin variety). In this case light travels as an electromagnetic wave along the fibre in a manner similar to that of microwave propagation along a waveguide. In order to have an idea of such mode of propagation we present at the end of this chapter a brief discussion on microwave propagation along a waveguide.

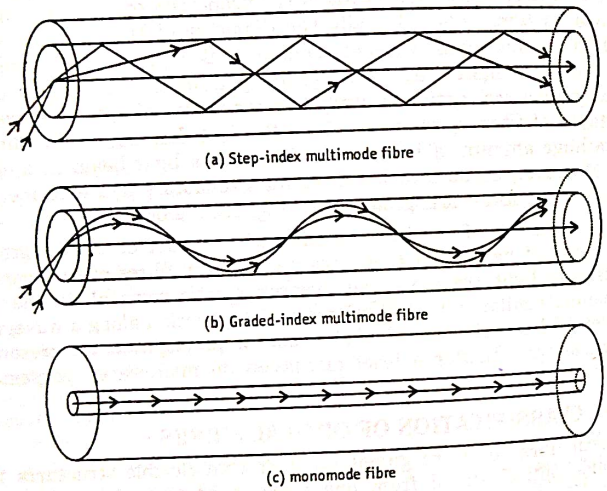
16.2 CLASSIFICATION OF OPTICAL FIBRES :

Optical fibres or fibre guides are hair thin flexible structures that can guide optical signal from one place to another. An optical fibre consists of an inner cylinder made of glass or plastic, called the *core*. The core is surrounded by a glass or plastic coating of lower refractive index called the *cladding*. The function of the cladding is to add mechanical strength to protect the core from environment and to reduce scattering losses at the core surface. Optical fibres can be classified on the basis of refractive index profile of the core and the number of modes propagating in the fibre.If the core has a uniform refractive index then it is called a *step-index fibre*. On the other hand, if the core has a non-uniform refractive index that gradually decreases from the centre toward the core-cladding interface, the fibre is classified as *graded-index fibre.*

Fig.16.2-1(a) shows a step-index fibre with relatively large core. Depending on the launch angle of the incident light into the fibre, there can be large number of different zig-zag ray paths or modes by which energy can travel down the core. Then it is called a *step-index multimode*

fibre. The rays travelling along different paths take different time to reach the other end of the fibre. This is known as *intermodal dispersion*. This difference in transit time leads to a distortion in the signal being transmitted.

Fig. 16.2-1(b) shows a *multimode graded index fibre* in which the problem of difference in transit times can be greatly reduced. Here the rays do not follow zig-zag paths but are refracted gradually. The refractive index is higher and hence velocity of light is smaller at the



(a) Step-index multimode fibre

(b) Graded-index multimode fibre

(c) monomode fibre

Fig. 16.2-1

central region.On the other hand, refractive index is smaller and velocity of light is greater near the cladding. So the rays spiralling around near the axis take shorter paths and travel slowly, whereas the rays spiralling around near the cladding take longer paths and travel at faster rate.The result is that all the rays tend to recross the axis at the same instant of time.

Fig. 16.2-1(c) shows a *monomode optical fibre*. Its core is made so narrow that only one mode of propagation is possible. The rays travel only parallel to the axis and the problem of intermodal dispersion is practically eliminated.

The multimode step-index fibre is the oldest of the three types. It is least expensive, rugged and can be easily infused with light. But it has the serious drawback of *intermodal dispersion* and is not suitable for long distance communication.

The multimode graded-index fibre has medium price. Intermodal dispersion problem is not very serious here. It is widely used in medium distance communications.

The monomode fibre is relatively expensive and requires laser sources. But here there is no intermodal dispersion problem. This type of fibre is most suitable for long distance communication.

If a large number of uncladded fibres are packed together to form bundle then light can leak through from one fibre to another. This is known as *cross-talk*. For this in today's fibre cladding is used. However, if a fibre is bent too sharply, the angle of incidence at the core-cladding interface may become less than the critical angle, a part of light escapes and may cause *cross-talk*.

If the fibres in a bundle are not aligned the bundle is said to be an *incoherent bundle*. On the other hand, if the fibres are properly aligned the bundle is said to be a *coherent bundle*.

16.3 ADVANTAGES OF OPTICAL FIBRES OVER COAXIAL CABLES OR TWISTED WIRE PAIRS :

Main application of optical fibres is in communication. It has the following advantages over the conventional coaxial cable or twisted wire pairs.

(i) The channel capacity of optical fibres is very large.

(ii) Losses are extremely low in optical fibres.

(iii) Optical fibres are small in size and light in weight.

(iv) Optical fibres are non-conductive, non-radiative and non-inductive. Thus there is practically no signal leakage, no cross-talk and the communication is very secured and free from extraneous electromagnetic disturbances.

(v) The basic raw material of fibres is silica which is abundantly available in nature whereas the basic raw material of coaxial cables is copper which is being increasingly costly.

(vi) Optical fibres are expected to be relatively cheap in near future.

16.4 APPLICATIONS OF OPTICAL FIBRES :

Optical fibres are nowadays being used in various fields. Some of the common applications are mentioned below.

(*i*) *Communication :* Most important application of optical fibres is in communication. Because of high bandwidth, low loss, small size, light weight and certain other attractive features optical fibres are being extensively used in both long distance communication and local area networks that include telephone, television, computers etc.

(*ii*) *Transmission of light :* Optical fibres are used for transmission of light to illuminate hard-to-reach places or to conduct light out of such places.

(*iii*) *Transmission of images :* A flexible bundle of optical fibres is called a *fibrescope*. It is extensibly used in carrying images. Light is passed down the outer fibres from the source and the reflected light is piped along the inner fibres.It is used in medicine to look into internal organs of human body and in non-destructive testing in industry.

(*iv*) *Sensors :* Optical fibres can be used in the field of fibre optic sensors which can sense and measure acoustic fields, magnetic fields, pressure, temperature etc.

(*v*) *Coupler :* Optical fibres can be used to couple two electrical circuits without introducing a direct link.

16.5 ACCEPTANCE ANGLE AND NUMERICAL APERTURE OF AN OPTICAL FIBRE :

We consider a step-index optical fibre with its core and cladding having refractive indices n_1 and n_2 respectively. Let n_0 be the refractive

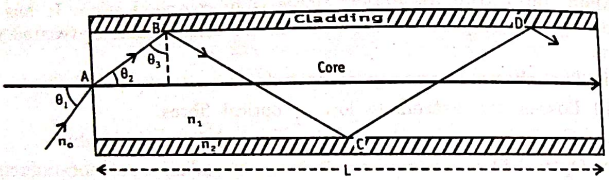


Fig. 16.5-1

index of the outside medium. Let a ray is incident on the entrance aperture of the fibre at an angle θ_1 with the axis. If θ_2 be the angle of refraction we get from Snell's law

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 \qquad ...(16.5\text{-}1)$$

In order to keep the light inside the core the angle of incidence θ_3 at the core-cladding interface must not be less than the critical angle θ_C. From Fig. 16.5-1,

$$\theta_3 = 90^\circ - \theta_2 \qquad ...(16.5\text{-}2)$$

If θ_1 is increased θ_2 increases and hence θ_3 decreases. So there is a maximum value θ_A of θ_1 for which θ_3 is not less than θ_C and the ray undergoes total internal reflection at the core-clading interface. This angle θ_A is known as *acceptance angle*. Thus the acceptance angle is the maximum angle of incidence for which any ray is totally internally reflected at the interface and, therefore, transmitted without loss. A cone of light of semiangle θ_A is known as acceptance cone. To determine $\theta_A$we write from Eqs. (16.5-1) and (16.5-2)

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 = n_1 \sin(90^\circ - \theta_3) = n_1 \cos\theta_3$$

For $\theta_1 = \theta_A$, $\theta_3 = \theta_C$, therefore,

$$n_0 \sin\theta_A = n_1 \cos\theta_C \qquad ...(16.5\text{-}3)$$

Since $\sin\theta_C = \dfrac{n_2}{n_1}$ we can write from Eq.(16.5-3)

$$n_0 \sin\theta_A = n_1\sqrt{1-\sin^2\theta_C} = n_1\sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2} \qquad ...(16.5\text{-}4)$$

The quantity $n_0 \sin\theta_A$ is defined as the **numerical aperture** (NA) of the fibre. All the rays entering inside a solid angle $\pi \sin^2\theta$ will be transmitted and hence the square of the numerical aperture gives a measure of the capacity of the fibre to transmit light power. If NA is large it would be easier to launch power into the fibre.

16.6 INTERMODAL DISPERSION IN A STEP-INDEX FIBRE :

Fig.16.5-1 shows a step-index fibre with the core and the cladding having uniform refractive indices n_1 and n_2 respectively. Depending on the launch angle there can be large number of different zig-zag paths along which light energy can travel down the fibre. The ray travelling along the axis takes the shortest time. If L is the length of the fibre then minimum time of travel is

$$t_{min} = \frac{\text{axial length}}{\text{Velocity of light in the fibre}} = \frac{L}{(c/n_1)} \qquad ...(16.6\text{-}1)$$

If a ray refracted into the fibre makes an angle θ_2 with the axis then path length l traversed by the ray will be $l = \dfrac{L}{\cos\theta_2}$. The path length will be the longest when the ray is incident on the core-cladding interface at the critical angle i.e, $\theta_3 = \theta_C$. Since $\theta_2 = 90^\circ - \theta_3$, the maximum time of travel is given by

$$t_{max} = \frac{l}{(c/n_1)} = \frac{Ln_1}{c\cos\theta_2} = \frac{Ln_1}{c\cos(90^\circ - \theta_C)} = \frac{Ln_1}{c\sin\theta_C}$$

Now $\sin\theta_C = \dfrac{n_2}{n_1}$ and therefore, $t_{max} = \dfrac{Ln_1^2}{cn_2}$ ...(16.6-2)

Thus the maximum time interval between the rays at the output end will be

$$\Delta t = t_{max} - t_{min} = \frac{Ln_1}{c}\left(\frac{n_1}{n_2} - 1\right) \qquad \ldots(16.6\text{-}3)$$

As the rays travel with a speed c/n_1 it will spread in space by a length

$$\Delta L = \frac{c}{n_1} \times \Delta t = L\left(\frac{n_1}{n_2} - 1\right) \qquad \ldots(16.6\text{-}4)$$

Informations to be transmitted through the fibre are usually digitally coded and sent along the fibre as a series of pulses. The difference in transit time has a harmful effect in the form of broadening the shape of pulses. To make the transmitted signal readable the time interval between two consecutive pulses must be large compared to the spread-out width Δt. Thus the intermodal dispersion limits the rate of signal transmission through the fibre.

16.7 RAY PATH IN A GRADED INDEX FIBRE :

In a graded index fibre the refractive index decreases smoothly as one moves from the axis radially outwards. Let the refractive index distribution may be represented as shown in Fig.16.7-1 by parabolic relation

$$n^2(r) = n_1^2\left[1 - \left(\frac{r}{\alpha}\right)^2\right], \quad |r| < r_0 \text{ (core region)}$$

$$n^2 = n_1^2\left[1 - \left(\frac{r_0}{\alpha}\right)^2\right], \quad |r| > r_0 \text{ (cladding region)} \qquad \ldots(16.7\text{-}1)$$

As the refractive index changes continuously a ray entering the fibre is continuously bent towards the axis of the fibre as shown in Fig.16.7-1. If the ray makes an angle $\theta(r)$ with the axis at any point on the surface

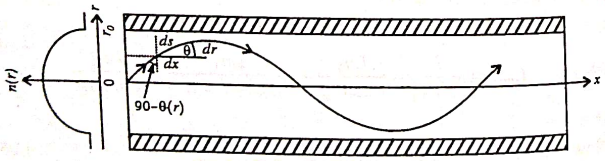


Fig. 16.7-1

of a cylinder of radius r, then angle of incidence is $90° - \theta(r)$ and hence from Snell's law $n(r) \sin [90° - \theta(r)]$ or, $n(r) \cos \theta(r)$ will be an invariant of the ray path.

Let
$$n(r) \cos \theta(r) = \beta \qquad \ldots(16.7\text{-}2)$$

Now if ds represents an infinitesimal arc length along the curve then

$$(ds)^2 = (dr)^2 + (dx)^2 \text{ or, } \left(\frac{ds}{dx}\right)^2 = \left(\frac{dr}{dx}\right)^2 + 1 \qquad \ldots(16.7\text{-}3)$$

From Fig. 16.7-1,

$$\frac{dx}{ds} = \cos\theta = \frac{\beta}{n(r)} \qquad \text{[using Eq. (16.7-2)]}$$

Therefore, from Eq. (16.7-3) we get

$$\left(\frac{dr}{dx}\right)^2 = \frac{n^2(r)}{\beta^2} - 1$$

Differentiating both sides with respect to x we get,

$$2\frac{dr}{dx}\cdot\frac{d^2r}{dx^2} = \frac{1}{\beta^2}\frac{dn^2}{dr}\cdot\frac{dr}{dx}$$

or,
$$\frac{d^2r}{dx^2} = \frac{1}{2\beta^2}\cdot\frac{dn^2}{dr}$$

Now using the expression for $n^2(r)$ for the core region as given by the Eq. (16.7-1) we can write

$$\frac{d^2r}{dx^2} = -\left(\frac{n_1}{\alpha\beta}\right)^2 . r \qquad \ldots(16.7\text{-}4)$$

Its solution is of the form

$$r(x) = A \sin(kx + \phi) \qquad \ldots(16.7\text{-}5)$$

where $k = \frac{n_1}{\alpha\beta}$

Suppose the ray is incident at $(x = 0, r = 0)$ making an angle θ_1 with x-axis. Then we may take $\phi = 0$, $\beta = n_1 \cos\theta_1$ and $k = \frac{1}{\alpha \cos\theta_1}$. To find the constant A in Eq. (16.7-5) we find that

$$\left.\frac{dr}{dx}\right|_{x=0} = \tan\theta_1 = Ak; \text{ or, } A = \frac{1}{k}\tan\theta_1 = \alpha \sin\theta_1$$

Thus the ray path in meridian plane is sinusoidal and can be written as

$$r(x) = A \sin kx$$

$$= \alpha \sin\theta_1 . \sin\left(\frac{x}{\alpha \cos\theta_1}\right) \qquad \ldots(16.7\text{-}6)$$

Acceptance angle :

The acceptance angle for meridional rays incident at the axial point can be found by considering the ray that after refraction travels along the interface between the core and the cladding as shown in Fig. 16.7-2.

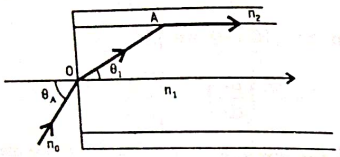


Fig. 16.7-2

Let the refractive index at the axis be n_1 and that of the cladding be n_2. Now application of Snell's law at O gives

$$n_0 \sin\theta_A = n_1 \sin\theta_1 \qquad \ldots(16.7\text{-}7)$$

Again from Snell's law $n(r)\cos\theta(r)$ is invariant along the ray path, where $\theta(r)$ is the angle made by the ray with the axis. Therefore, for the points O and A,

$$n_1 \cos\theta_1 = n_2 \cos 0°; \text{ or, } \cos\theta_1 = \frac{n_2}{n_1}$$

$$\therefore \quad n_0 \sin\theta_A = n_1\sqrt{1-\cos^2\theta_1} = n_1\sqrt{1-\frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2} \qquad \ldots(16.7\text{-}8)$$

Here it is important to point out that Eq. (16.7-8) is valid only for meridional rays incident at axis. For other rays the value of θ_A may not agree with that given by the Eq. (16.7-8).

16.8 WAVEGUIDE :

A waveguide is a structure consisting of highly conducting metallic walls and dielectrics that is used to guide electromagnetic waves in the microwave region (3 - 300 GHz) from one place to another. It is usually in the form of a hollow metallic pipe of rectangular or circular cross-section. A wire antenna is often used to excite electromagnetic waves inside the waveguide. For effective transmission of power through a guide its transverse dimension must be comparable to the wavelength of the wave.The electromagnetic wave launched into the waveguide suffers multiple internal reflections during its course of propagation

through the guide.The resulting types and modes of propagation through the guide can be obtained by solving Maxwell's equations with appropriate boundary conditions. For simplicity we consider a rectangular waveguide with air as dielectric inside. Then Maxwell's equations for the region inside the waveguide are,

$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \ldots(16.8\text{-}1)$$

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad \ldots(16.8\text{-}2)$$

$$\vec{\nabla}.\vec{E} = 0 \qquad \ldots(16.8\text{-}3)$$

$$\vec{\nabla}.\vec{H} = 0 \qquad \ldots(16.8\text{-}4)$$

where the symbols have their usual meanings.

Let the waves to be propagated be monochromatic with time dependence $e^{-i\omega t}$. Then the Eqs. (16.8-1) and (16.8-2) take the form

$$\vec{\nabla}\times\vec{H} = -i\omega\varepsilon_0 \vec{E} \qquad \ldots(16.8\text{-}5)$$

$$\vec{\nabla}\times\vec{E} = i\omega\mu_0 \vec{H} \qquad \ldots(16.8\text{-}6)$$

Taking curl of Eq. (16.8-6) and using Eq.(16.8-5) we can show that $\vec{E}$ satifies the wave equation

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{E} = 0 \qquad \ldots(16.8\text{-}7)$$

where $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$ is the velocity of light in free space.

Similarly we can show that $\vec{H}$ also satisfies the wave equation. Here we are interested in the solution that represents waves propagating along the axis of the guide (taken as z-axis). Thus we seek solution of the form

$$\vec{E}(x,y,z,t) = \vec{E}_0(x,y)\, e^{i(kz-\omega t)}$$

$$\vec{H}(x,y,z,t) = \vec{H}_0(x,y)e^{i(kz-\omega t)} \qquad \ldots(16.8\text{-}8)$$

Eq. (16.8-5) may be written in component forms as

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega\varepsilon_0 E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0 E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega\varepsilon_0 E_z$$

Using the solution (16.8-8) we get

$$\frac{\partial H_{oz}}{\partial y} - -ik.H_{oy} = -i\omega\varepsilon_0 E_{ox} \qquad \ldots(16.8\text{-}9)$$

$$ikH_{ox} - \frac{\partial H_{oz}}{\partial x} = -i\omega\varepsilon_0 E_{oy} \qquad \ldots(16.8\text{-}10)$$

$$\frac{\partial H_{oy}}{\partial x} - \frac{\partial H_{ox}}{\partial y} = -i\omega\varepsilon_0 E_{oz} \qquad \ldots(16.8\text{-}11)$$

Similarly from Eq. (16.8-6) we can have

$$\frac{\partial E_{oz}}{\partial y} - ikE_{oy} = i\omega\mu_0 H_{ox} \qquad \ldots(16.8\text{-}12)$$

$$ikE_{ox} - \frac{\partial E_{oz}}{\partial x} = i\omega\mu_0 H_{oy} \qquad \ldots(16.8\text{-}13)$$

$$\frac{\partial E_{oy}}{\partial x} - \frac{\partial E_{ox}}{\partial y} = i\omega\mu_0 H_{oz} \qquad \ldots(16.8\text{-}14)$$

Eliminating E_{oy} from (16.8-10) and (16.8-12) we get

$$H_{ox} = \frac{i}{h^2}\left[k\frac{\partial H_{oz}}{\partial x} - \omega\varepsilon_0\frac{\partial E_{oz}}{\partial y}\right] \qquad \ldots(16.8\text{-}15)$$

where $h^2 = \dfrac{\omega^2}{c^2} - k^2$

Similarly considering other equations appropriately we may have

$$H_{oy} = \frac{i}{h^2}\left[k\frac{\partial H_{oz}}{\partial y} + \omega\varepsilon_0\frac{\partial E_{oz}}{\partial x}\right] \qquad \ldots(16.8\text{-}16)$$

$$E_{ox} = \frac{i}{h^2}\left[k\frac{\partial E_{oz}}{\partial x} + \omega\mu_0\frac{\partial H_{oz}}{\partial y}\right] \qquad \ldots(16.8\text{-}17)$$

$$E_{oy} = \frac{i}{h^2}\left[k\frac{\partial E_{oz}}{\partial x} - \omega\mu_0\frac{\partial H_{oz}}{\partial y}\right] \qquad \ldots(16.8\text{-}18)$$

Thus we find that the transverse components are being completely determined by the longitudinal components i.e., the problems of determining the fields inside the waveguide reduces to that of determining the longitudinal field components only. In general both E_{oz} and H_{oz} are present. However, we shall consider two separate situations.

Case I—

If $E_{oz} = 0$ then entire field is determined by H_{oz}. For this corresponding waves are called **H-waves**. Since only transverse electric field exists the corresponding wave is also called transverse electric or **TE waves**.

Case II—

If $H_{oz} = 0$ then entire field is determined by E_{oz}. For this it is called **E-waves**. Since only transverse magnetic field exists the corresponding wave is also called transverse magnetic or **TM waves.**

There may be a third choice where both E_{oz} and H_{oz} are zero. Corresponding waves will be called transverse electro-magnetic or **TEM waves**. But Eqs. (16.8-15) – (16.8-18) show that in this case the entire field inside the waveguide vanishes identically. This indicates that *pure TEM waves cannot exist in a rectangular waveguide.*

TE modes :

In this case $E_{oz} = 0$ and the entire field is determined by H_{oz}. The wave equation satisfied by $\vec{H}$ is

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\vec{H} = 0$$

Since the z-dependence of $\vec{H}$ is assumed to be of the form e^{ikz}, the above equation can be written for the z-component of $\vec{H}$ as

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+h^2\right)H_{oz}=0 \qquad \ldots(16.8\text{-}19)$$

where $h^2=\frac{\omega^2}{c^2}-k^2$.

We assume a solution of the form

$$H_{oz}(x,y)=F_1(x).F_2(y)$$

Substituting it in Eq. (16.8-19) and dividing throughout by F_1F_2 we get

$$\frac{1}{F_1}\cdot\frac{d^2F_1}{dx^2}+\frac{1}{F_2}\cdot\frac{d^2F_2}{dy^2}=-h^2$$

Let, $\quad \frac{1}{F_1}\cdot\frac{d^2F_1}{dx^2}=-k_x^{\,2}$ and $\frac{1}{F_2}\cdot\frac{d^2F_2}{dy^2}=-k_y^{\,2}$

where $h^2=k_x^{\,2}+k_y^{\,2}$

Thus the final solution is of the form

$$H_{oz}(x,y)=(A\cos k_x x+B\sin k_x x)\,(C\cos k_y y+D\sin k_y y) \qquad \ldots(16.8\text{-}20)$$

where the constants A, B, C and D are to be found out from boundary conditions. We consider a rectangular waveguide of sides a and b and made of perfectly conducting walls. The boundary conditions are

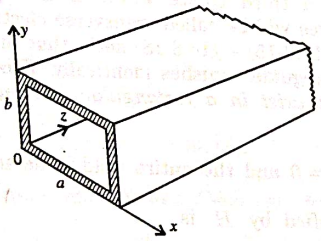


Fig. 16.8-1

$E_y=0$ at $x=0$ and $x=a$

$E_x=0$ at $y=0$ and $y=b$

Now from Eq. (16.8-18),

$$E_{oy}\propto\frac{\partial H_{oz}}{\partial x}=0 \quad \text{at } x=0$$

This condition when applied to Eq. (16.8-20) gives $B=0$.

From Eq. (16.8-17),

$$E_{ox}\propto\frac{\partial H_{oz}}{\partial y}=0 \quad \text{at } y=0$$

Application of this condition to Eq. (16.8-20) gives $D=0$

Thus, $\quad H_{oz}(x,y)=H_o\cos k_x x.\cos k_y y \qquad \ldots(16.8\text{-}21)$

where $H_o=AC$ is determined by the introduced field.

Now the condition

$$E_{oy}\propto\frac{\partial H_{oz}}{\partial x}=0 \quad \text{at } x=a$$

indicates that $\sin k_x a=0$ or, $k_x=\frac{m\pi}{a}$; $m=0,\ 1,\ 2,\ 3\ldots$

Similarly the condition

$$E_{ox}\propto\frac{\partial H_{oz}}{\partial y}=0 \quad \text{at } y=b$$

gives $\sin k_y b=0$ or, $k_y=\frac{n\pi}{b}$; $n=0,\ 1,\ 2,\ 3\ \ldots$

Thus $\quad H_{oz}=H_o\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad \ldots(16.8\text{-}22)$

Now as H_{oz} is known, other field components can be easily calculated by using the relations (16.8-15) – (16.8-18).

Now $\quad h^2=k_x^2+k_y^2$

or, $\quad \frac{\omega^2}{c^2}-k^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$

or, $k^2=\frac{\omega^2}{c^2}-\left\{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right\}$ $\qquad \ldots(16.8\text{-}23)$

If $\omega<c\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ then k is imaginary and the fields get attenuated and cannot propagate through the waveguide. On the other hand, if $\omega>c\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ then k is real and the waves can

propagate through the waveguide without attenuation. Thus there is a critical frequency ω_c, called *cut-off frequency* such that all frequency components below this frequency cannot propagate through the waveguide. ω_c is given by

$$\omega_c = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad \ldots(16.8\text{-}24)$$

The cut-off wavelength λ_c is given by

$$\lambda_c = \frac{c}{\omega_c/2\pi} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \qquad \ldots(16.8\text{-}25)$$

Propagation is possible only when wavelength $\lambda < \lambda_c$. The value of cut-off wavelength λ_c depends on the transverse dimension of the waveguide and also on the mode (m,n) of propagation. $(TE)_{00}$ mode is not possible because for $m = n = 0$ all the field components vanish. For $m = 0$, $n = 1$ we have $(\lambda_c)_{01} = 2b$. For $m = 1$, $n = 0$ we have $(\lambda_c)_{10} = 2a$. A mode for which cut-off wavelength is highest is called *dominant mode*. Thus assuming $a > b$, we find that $(TE)_{10}$ is the dominant mode.

We have the propagation constant k given by Eqs. (16.8-23) and (16.8-24) as,

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_c^2}{c^2} \qquad \ldots(16.8\text{-}26)$$

Now we define guide wavelength as

$$\lambda_g = \frac{2\pi}{k}$$

Since free space wavelength $\lambda = \frac{c}{\omega/2\pi}$, we can write from Eq. (16.8-26)

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \quad \text{or,} \quad \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \qquad \ldots(18.6\text{-}27)$$

Thus λ_g is greater than corresponding free space wavelength. The phase velocity,

$$v_p = \frac{\omega}{k} = \frac{\omega}{\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_c^2}{c^2}}} = \frac{c}{\sqrt{1 - (\lambda/\lambda_c)^2}} \qquad \ldots(16.8\text{-}28)$$

So the phase velocity of propagation through the guide is greater than the phase velocity c in the free space. This does not violate the principle of relativity because the group velocity

$$v_g = \frac{d\omega}{dk} = c\sqrt{1 - (\lambda/\lambda_c)^2} \qquad \ldots(16.8\text{-}29)$$

which determines the velocity of energy propagation and is always less than c. Actually the zig-zag paths followed by the waves make the group velocity smaller than c. Near the cut-off $\lambda = \lambda_c$, $v_p \to \infty$ and $v_g \to 0$. Note that $v_p v_g = c^2$.

TM modes :

In this case $H_{oz} = 0$ and the entire field is determined by E_{oz}. Proceeding as before we can show that

$$E_{oz} = E_o \sin\left(\frac{m\pi x}{a}\right) . \sin\left(\frac{n\pi y}{b}\right) \qquad \ldots(16.8\text{-}30)$$

In this case all the field components vanish if both m and n or any one of them is zero. Thus $(TM)_{00}$, $(TM)_{01}$ or $(TM)_{10}$ modes are not possible. For TM mode highest wavelength available for transmission through the guide is

$$(\lambda_c)_{11} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}}$$

which is definitely less than that for TE waves where the longest wavelength is

$$(\lambda_c)_{10} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2}} \qquad \text{(assuming } a > b\text{)}$$

Thus frequency band available for transmission through the waveguide is greater in TE modes than that in TM modes.

Applications :

Waveguides are used to transmit electromagnetic energy in microwave region over short distances. In radar they are used for transmission of high frequency power from transmitter to antenna and from antenna to the receiver.

SOLVED PROBLEMS

1. A step index fibre has a core of refractive index 1.55 and a cladding of refractive index 1.53. Determine its numerical aperture and acceptance angle.

Ans. Numerical aperture $= n_0 \sin\theta_A = \sqrt{n_1^2 - n_2^2}$

$= \sqrt{1.55^2 - 1.53^2}$

$= 0.248$

If surrounding medium is air $n_0 = 1$ and the acceptance angle $\theta_A = \sin^{-1}(0.248) = 14.37°$.

2. A step index fibre has a core of refractive index 1.50 and a cladding of refractive index 1.40. If the fibre is used in a water environment find its numerical aperture and the acceptance angle. Take the refractive index of water as 1.33.

Ans. $NA = n_0 \sin\theta_A = \sqrt{n_1^2 - n_2^2} = \sqrt{1.50^2 - 1.40^2} = 0.538$

$$\theta_A = \sin^{-1}\frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \sin^{-1}\frac{\sqrt{1.50^2 - 1.40^2}}{1.33}$$

$= 23.9°$

3. A step index fibre has a core of refractive index 1.500 and a cladding of refractive index 1.489. Calculate the temporal broadening of a pulse in ns/km. Also, calculate the spread in space for a length of 1 km of the fibre.

Ans. $\Delta t = \frac{Ln_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{1000 \times 1.500}{3 \times 10^8}\left(\frac{1.500}{1.489} - 1\right)$ s/km.

$= 36.9$ ns/km.

$$\Delta L = \Delta t \times \frac{c}{n_1} = L\left(\frac{n_1}{n_2} - 1\right) = 1000\left(\frac{1.500}{1.489} - 1\right) \text{m} = 7.4 \text{ m}$$

4. Consider an optical fibre of diameter 50 μm. If the core is of refractive index 1.6 find the number of total internal reflections that a ray incident at an angle 30° will suffer in travelling through 1 m of the fibre.

Ans. Fig. 16.P-1 shows a section of a fibre with relevant angles. If D is the diameter, θ is the angle of incidence and θ_1 is the angle of refraction then

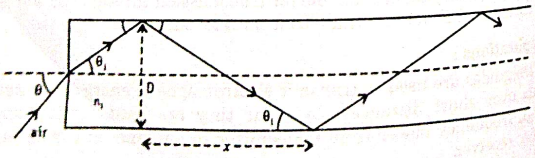


Fig. 16.P-1

the axial distance x travelled by the ray between two consecutive reflections is given by

$$\frac{D}{x} = \tan\theta_1 \text{ ; or, } x = \frac{D}{\tan\theta_1} \quad \ldots(1)$$

Now from Snell's law,

$$\frac{\sin\theta}{\sin\theta_1} = n_1 \text{ ; or, } \sin\theta_1 = \frac{\sin\theta}{n_1} \quad \ldots(2)$$

$$\therefore \quad x = \frac{D\cos\theta_1}{\sin\theta_1} = \frac{D\sqrt{1 - \frac{\sin^2\theta}{n_1^2}}}{\sin\theta/n_1} = \frac{D\sqrt{n_1^2 - \sin^2\theta}}{\sin\theta}$$

If L be the length of the fibre then the number of reflections will be

$$N = \frac{L}{x} = \frac{L\sin\theta}{D\sqrt{n_1^2 - \sin^2\theta}}$$

$$= \frac{1 \times 1/2}{50 \times 10^{-6}\sqrt{1.6^2 - \frac{1}{4}}} \approx 6580$$

5. Find the cut-off wavelength for a rectangular waveguide of dimension 5 cm × 2.5 cm for $(TE)_{10}$ mode. Also, calculate the group velocity of propagation along the waveguide for this mode at a frequency of 6 GHz.

Ans. $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$ or, $(\lambda_c)_{10} = \frac{2}{\sqrt{\left(\frac{1}{5}\right)^2}} = 10 \text{ cm} = 0.1 \text{ m}$.

$$v_g = c\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$$

where $\lambda_c = 0.1$ m and $\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05$ m

$$\therefore \quad v_g = c\sqrt{1 - (0.5)^2} = 0.866\, c$$

QUESTIONS

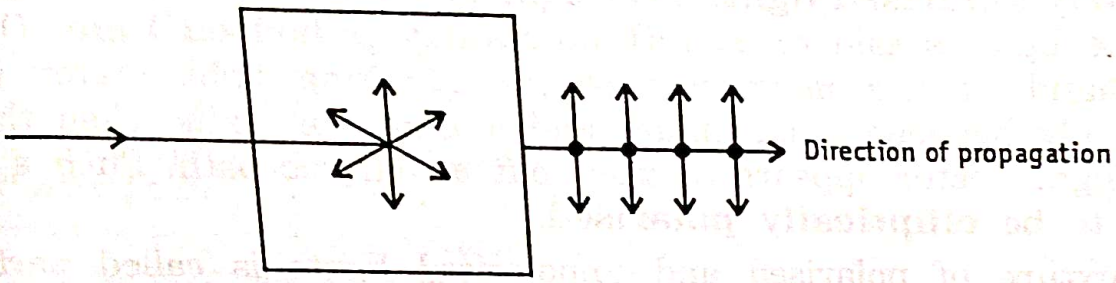
1. What is an optical fibre? Describe the working principle of an optical fibre. What do you mean by 'cross-talk'?
2. What is meant by acceptance angle and numerical aperture of a fibre? Obtain an expression for numerical aperture of a step-index fibre.

CHAPTER 13

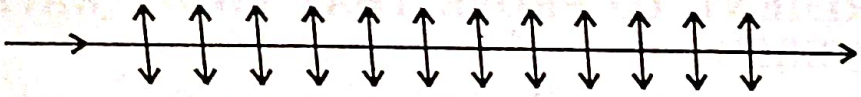
POLARISATION OF LIGHT

13.1 INTRODUCTION :

The phenomena of interference and diffraction of light show that ıe light is a wave motion which may be longitudinal or transverse ı nature. It is the phenomenon of polarisation which for its explanation equires that the light must be a transverse wave. In fact in the lectromagnetic theory (Chap. 14) light is considered as a transverse lectromagnetic wave consisting of vibrating electric and magnetic ectors at right angles to each other and also at right angles to the



(a) Unpolarised light



(b) Plane polarised light with its vibration in the plane of the paper

(c) Plane polarised light with its vibration in the plane perpendicular to the plane of the paper

Fig. 13.1-1

direction of propagation of light. The electric vector is called **light vector** because it is responsible for the sensation of vision. In an ordinary beam of light with millions of waves the light vectors of component waves will remain in all possible directions on a plane drawn at right angles to the direction of propagation. This happens so due to random orientations of excited atoms or molecules in the source. Such an ordinary beam of light with the electric vectors arranged symmetrically about the direction of propagation is called an *unpolarised light*. We may assume all these vibrations to be resolved into two rectangular components of equal amplitude but having a relative phase difference which varies rapidly and randomly. Now, if by some means one of these rectangular vibrations is cut off we get vibrations of all the component waves confined in one definite direction. Such a light is said to be *plane*

polarised light (also called *p-state light*). The pictorial representations of unpolarised and plane polarised lights are shown in Fig.13.1-1.

The electric vector of a plane polarised light is confined to a particular plane, known as the **plane of vibration**. This plane contains the $\vec{E}$-vector and the propagation vector $\vec{k}$. The plane perpendicular to the plane of vibration is known as **plane of polarisation**.

By the superposition of two plane polarised waves under suitable conditions the resultant light vector may be made to rotate in a plane perpendicular to the direction of propagation. If the magnitude of the resulting light vector remains constant the tip of the light vector appears to trace out a circle at a fixed space. Such a light is said to be **circularly polarised**. If, on looking towards the incoming light the resultant light vector appears to rotate clockwise, then the light is said to be **right circularly polarised light**. If the light vector rotates counter-clockwise then the light is said to be **left circularly polarised light**. On the other hand, if the magnitude of the resulting light vector varies periodically between a maximum and a minimum value then the tip of the light vector appears to trace out an elliptic path. Such a light is said to be **elliptically polarised**.

A mixture of polarised and unpolarised lights is called partially polarised light.

13.2 EXPERIMENT TO PROVE THAT LIGHT WAVES ARE TRANSVERSE :

When ordinary or common light from a source O is made to pass successively through two tourmaline crystals R_1 and R_2 with their axes parallel, light will be received by the eye E (Fig. 13.2-1).

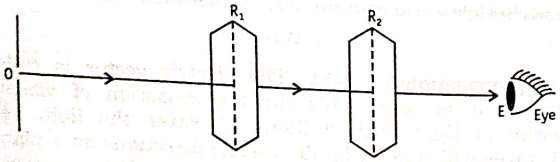


Fig. 13.2-1

If now the axis of R_2 be rotated with the incident ray as axis, the intensity of the light which is transmitted through R_2 will gradually decrease. No light will be received by the eye when the axis of R_2 becomes perpendicular to that of R_1.

If we assume that the waves of light are transverse, then the vibrations of the light falling normally on the surface of the first crystal R_1will lie in all possible directions on its surface. These vibrations will then be resolved into two perpendicular directions, one parallel to the

axis of R_1 and another perpendicular to it. The crystal R_1 will allow only those vibrations to pass through it which are parallel to its axis while the vibrations perpendicular to its axis are cut off. If the axis of R_2 be kept parallel to that of R_1, then the vibrations from R_1 will also pass through R_2 and the eye will receive light. If the axis of R_2 be kept perpendicular to that of R_1, then the vibrations from R_1 now become perpendicular to the axis of R_2. The crystal R_2, therefore, intercepts the vibrations from R_1 and the eye receives no light at all.

This experiment shows that the light waves are transverse; otherwise the light coming out of R_1 could never be extinguished by simply rotating the crystal R_2.

13.3 POLARISATION BY REFLECTION AND BREWSTER'S LAW:

If an ordinary beam of light is incident on a glass plate at a particular angle $i_p \approx 57°$ it is found that the reflected light becomes almost wholly a plane polarised light, the plane of vibration being perpendicular to the plane of incidence. For angles other than i_p the reflected light is partially polarised. This particular angle of incidence at which the reflected light is almost wholly polarised is called the **angle of polarisation** for the given surface. It depends on the nature of the reflecting medium and the wavelength of light.

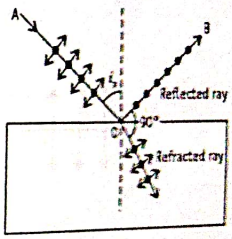


Fig. 13.3-1

Brewster discovered that at the polarising angle the reflected and the refracted rays are just 90° apart. Therefore, from Fig. 13.3-1, the angle of refaction is $90° - i_p$. Hence the refractive index of glass with respect to air is,

$$n = \frac{\sin i_p}{\sin(90° - i_p)} = \tan i_p \qquad \ldots(13.3\text{-}1)$$

Thus the tangent of the angle of polarisation is equal to the refractive index of the reflecting medium. This is known as **Brewster's law.**

Explanation :

The light vectors in the unpolarised incident light beam may be resolved into two rectangular components— one parallel to the plane of incidence (AOB) and the other perpendicular to the plane of incidence. At the polarising angle, since the reflected and refracted rays are 90° apart, at the point of incidence O the vibrations (shown by parallel arrow heads in Fig. 13.3-1) which are in the plane of incidence become parallel to the direction of reflection OB. As light is a transverse wave they cannot get reflected along OB. However, the other vibrations

perpendicular to the plane of incidence (shown by dots in Fig. 13.3-1) can be reflected partly.

13.4 POLARISATION BY REFRACTION :

If a beam of ordinary light is incident on a glass plate at the polarising angle i_p for glass, the transmitted light will be partially plane polarised though the reflected light will be almost wholly plane polarised. If a number of parallel glass plates are used then at each reflection, the rays whose vibrations are perpendicular to the plane of incidence are reflected. The transmitted light contains vibrations in the plane of incidence as well as a reduced amount of vibrations perpendicular to

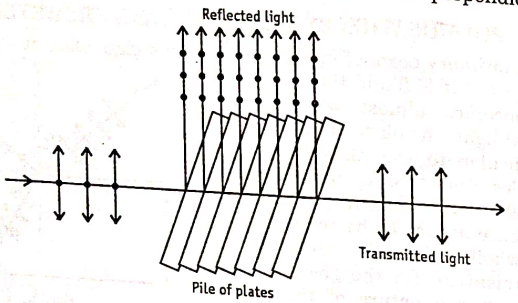


Fig. 13.4-1

the plane of incidence.Thus with a large number of parallel plates the transmitted light loses more and more of its perpendicular vibrations at each reflection and ultimately becomes wholly polarised whose vibrations remain in the plane of incidence. This arrangement is known as **piles of plates** which is shown in Fig.13.4-1.

13.5 GEOMETRY OF CALCITE CRYSTAL AND MEANING OF ITS OPTIC AXIS AND PRINCIPAL SECTION :

Iceland spar is the transparent variety of calcite($CaCO_3$) which crytallises in many forms. Like all other crystals, it is also bounded by the plane faces which are inclined to one another by fixed angles. When this crystal is struck, it breakes obliquely in three different planes forming a rhombohedral body like $ABCDA_1B_1C_1D_1$ as showm in Fig. 13.5-1. This operation of breaking a crystal (quartz is an exception) across definite planes is known as *cleavage* and the faces produced after cleavage are known as cleavage faces. The rhomb of iceland spar is bounded by six parallelogram faces, the angles of each of which are 101°55′ and 78°5′.

(a) Optic axis :

(*i*) On examination of the rhomb $ABCDA_1B_1C_1D_1$ of an iceland spar, we shall find that each of its eight corners is formed by the meeting of three plane faces. The two diagonally opposite corners A and C_1 of the rhomb (Fig.13.5-1), are formed by the meeting of three plane faces, each of them having an obtuse angle (101°55′) at A and C_1. If a straight line be drawn within the crystal either from A or from C_1, equally inclined to the three meeting faces at A or C_1 then this straight line or any other straight line parallel to it will be called as the *optic axis* of the crystal. Thus optic axis of a crystal is a *direction* and *not a straight line.*

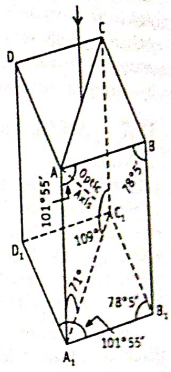


Fig. 13.5-1

(*ii*) *Optic axis* of a crystal is defined as the *direction* through it, along which if a ray travels then there will be no *double refraction* of the ray and both the ordinary and extraordinary rays travel with *equal velocity* along this direction.

A doubly refracting crystal possessing only one optic axis is called a *uniaxial crystal.*

(b) Principal section :

Principal section of a crystal is its section by a plane which passes through the optic axis of the crystal and is perpendicular to its two opposite refracting faces.

Since there are infinite number of lines parallel to the direction of the optic axis, there are also an infinite number of principal sections, of which one such section ACC_1A_1 is shown in Fig. 13.5-1.

13.6 DOUBLE REFRACTION :

Erasmus Bartholinus discovered (in 1669) that when an ordinary ray of unpolarised light is made incident on a crystal like calcite, quartz etc. it is refracted into two rays. One of these rays obeys the laws of

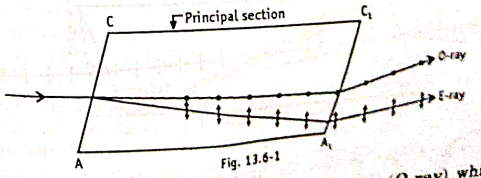


Fig. 13.6-1

refraction of light and is called the **ordinary ray** (*O*-ray) while the other ray does not, in general, obey those laws and is called the

extraordinary ray (*E*-ray) (Fig. 13.6-1). Both of these *E*- and *O*-rays are plane polarised whose vibrations are along and at right angles to the principal section. This phenomenon in which a single incident ray is refracted into two rays is called *double refraction* or *birefringence* and the crystals which exhibit the phenomenon are called *doubly refracting crystals* or *birefringent*. This phenomenon is exhibited by other crystals excepting those which have cubic arrangements of their atoms. The state of polarisation of the refracted rays can be determined by using a tourmaline crystal.

13.7 NICOL PRISM :

Nicol prism is an optical device designed from calcite and is used for the production and analysis of plane polarised light. A beam of ordinary light entering the calcite crystal breaks up into *E*-rays and *O*-rays by double refraction. The *O*-ray is cut off by total internal refraction while the *E*-ray is allowed to pass through.

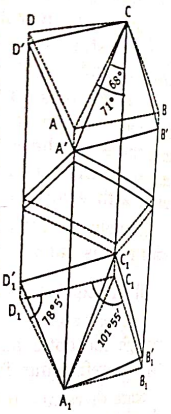


Fig. 13.7-1

To construct a Nicol prism a calcite rhombohedron of length three times its breadth is produced from natural crystal by cleavage. The end faces are then cut artificially so that the acute angles of the principal section is reduced from 71° to 68° (Fig. 13.7-1).

The resulting crystal is then cut into two portions by a plane perpendicular to both the principal section and the end faces of the rhomb. The cut surfaces are highly polished and cemented together by a layer of Canada-balsam. The sides of the prism are coated with lamp-black and are kept covered by a brass tube. The two end

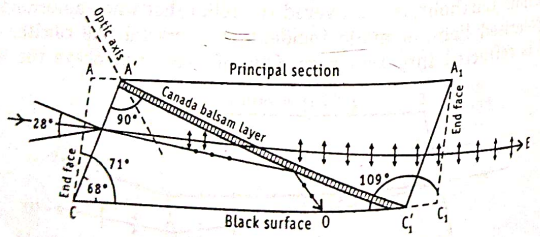


Fig. 13.7-2

faces are kept open so that the light may be incident on one end face and may emerge out of the other end face.

A section of the resulting crystal parallel to the principal section is shown in Fig 13.7-2.

Action of Nicol as polariser :

An incident ray falling on one of the end faces of the Nicol and moving in a direction nearly parallel to its length is divided into ordinary and extraordinary rays whose vibrations are respectively perpendicular and parallel to the principal section of the Nicol. Canada-balsam is optically rarer than calcite for *O*-ray. The geometry of Nicol prism is such that the *O*-ray is incident on balsam layer at an angle greater than the critical angle and is, therefore, totally reflected. This totally reflected light pass out through the side of the prism and is absorbed by the lamp-black layer on the sides of the prism.

For extraordinary ray, Canada-balsam is optically denser than calcite and this ray is freely transmitted and emerges out of the opposite end face. Thus we get a plane polarised light whose vibrations are parallel to the principal section.

For total reflection of the ordinary ray, the angle of incidence at the balsam layer must be greater than the critical angle and this will happen when the angle of incidence of the ray on the end face does not exceed 14°. Again the refractive index of calcite for *E*-ray depends upon the direction of propagation through the crystal. Hence for *E*-ray, the balsam layer will behave as an optically less dense medium than calcite, when the angle of incidence at the end face exceeds a certain limit. Thus to avoid the transmission of *O*-ray and total reflection of *E*-ray, the angle between the extreme rays of the incoming beam is limited to about 28°.

Action of Nicol as analyser :

Nicol prism can also be employed as analyser of polarised light.

Suppose a plane polarised light be made incident on one face of a Nicol so that the direction of its vibration of amplitude a, makes an angle θ with the principal section of the Nicol.This vibration of amplitude a will be resolved into two vibrations of amplitudes $a\cos\theta$ and $a\sin\theta$ lying respectively parallel and perpendicular to the principal section of the Nicol. The component $a\cos\theta$, which is parallel to the principal section, will be freely transmitted as *E*-ray, while the component $a\sin\theta$ which is perpendicular to the principal section of Nicol, will behave as the *O*-ray and will be cut off by total reflection from the balsam layer.

If now the Nicol is rotated with the incident ray as axis, the value of θ will change. When θ is zero (i.e., when the principal section of Nicol is parallel to the vibration of incident light) the amplitude of transmitted component will be a, which is maximum, and the field would be brightest. But when $\theta = 90°$ (i.e., when the principal section of Nicol is perpendicular to the vibration of incident light) the amplitude of the transmitted component is zero and the field would be dark.

Thus we conclude that if the incident light be plane polarised, the intensity of light transmitted through the Nicol would be greatest in its one position while that would be zero in another position which is at right angle to the former. At this position of Nicol, in which the field is dark, the plane of vibration of the incident plane polarised light would be perpendicular to the principal section of the Nicol.

If the incident light be unpolarised, then for any position of the Nicol, the vibrations of the incident light can be resolved parallel and perpendicular to its principal section. The parallel component will pass through the Nicol, while the perpendicular component will be refused transmission and the field of the Nicol will always remain equally bright.

Parallel and crossed Nicols :

Nicol prism can be used both as polariser and analyser of light beam. Two Nicols are placed coaxially as shown in Fig. 13.7-3. The first Nicol P producing plane polarised light is called the polariser, while the second Nicol A which analyses the incident light is called analyser. When the principal sections of the two Nicols are parallel, the vibrations of E-ray coming out of the polariser are in the principal section of both the

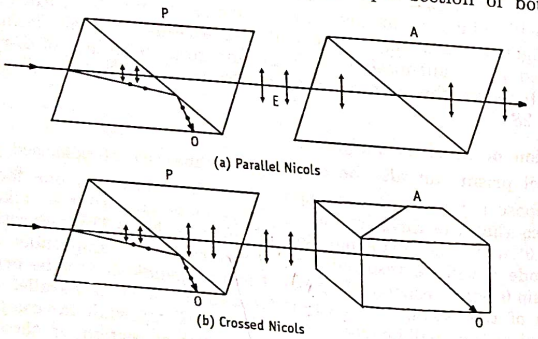


Fig. 13.7-3

polariser and the analyser. As a result it transmits light freely as E-ray through the analyser A. Such arrangement is called **parallel Nicols**.

If the analyser is gradually rotated the intensity of the transmitted light decreases and when their principal sections become mutually perpendicular, no light is transmitted. The vibrations of the E-ray which are in the principal section of polariser becomes perpendicular to principal section of the analyser. The E-ray from polariser becomes O-ray in the analyser and is totally reflected. Nicols in such arrangement are said to be **crossed Nicols**.

13.8 MALUS'S LAW :

Malus experimentally obtain a relation which shows how the intensity of light transmitted by the analyser varies with the angle that its plane of transmission makes with that of the polariser. Suppose that the angle between the two planes of transmission be θ at any instant, as shown in Fig. 13.8-1. The light vector $AP = E_0$ in the plane polarised light emerging from the polariser may be resolved into two components $AE = E_0 \cos\theta$ and $AO = E_0 \sin\theta$ respectively, along and perpendicular to the plane of transmission of the analyser. The perpendicular component $E_0 \sin\theta$ is eliminated while the parallel component is freely transmitted through the analyser. Hence the intensity of light that emerges from the analyser is given by,

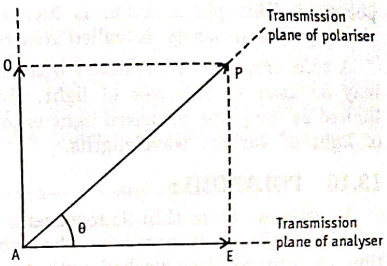


Fig. 13.8-1

$$I = E_0^2 \cos^2\theta = I_0 \cos^2\theta \qquad \ldots(13.8\text{-}1)$$

where I_0 is the intensity of the polarised light incident on the analyser. This is, of course, one-half of the intensity of the unpolarised light incident on the polariser. Eq. (13.8-1) shows that the transmitted intensity varies as the square of the cosine of the angle between the planes of transmission of polariser and analyser. This is known as **Malus's law**.

13.9 POLARISATION BY DICHROIC CRYSTALS :

There are certain doubly refracting crystals (like tourmaline) which not only produce O-rays and E-rays but also absorb one of these two

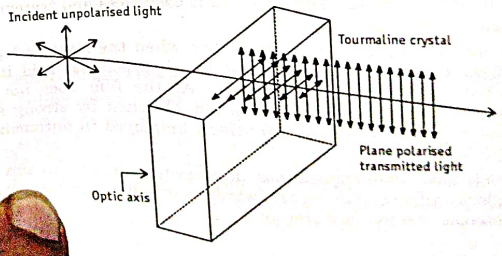


Fig 13.9-1

rays much more strongly than the other. Thus if an unpolarised light is passed through a crystal like tourmaline of proper thickness one of the components is completely absorbed and the other is transmitted in appreciable amount (Fig. 13.9-1). The emergent beam will be linearly polarised. This phenomenon is known as *dichroism* and the crystals showing this property is called *dichroic crystals*.

A thin crystal of tourmaline with its faces cut parallel to optic axis may be used as polariser of light. However, its use as a polariser is limited as the plane polarised light is coloured due to unequal absorption of light of various wavelengths.

13.10 POLAROID :

A polaroid is a thin transparent film which can porduce plane polarised light on transmission through it. It usually consists of a thin film of nitrocellulose packed with tiny dichroic crystals (herapathite) with their optic axes all parallel. Because of certain advantages it has largely replaced Nicol prism for producing and analysing polarised light in the laboratory.

At first a paste of small crystals of herapathite (Iodo-sulphate of quinine) is prepared in nitrocellulose. This paste is then squeezed out through a fine slit. As a result only those crystals pass whose axes are parallel to the length of the slit. In this way a large fine sheet is produced which contains millions of tiny crystals with their optic axes all parallel. This is mounted between two thin sheets of glass forming what is known as polaroid.

In more recent developments a better method was found in which individual molecules, rather than micro-crystals were employed to construct a polaroid known as ***H*-polaroid**. In this process a sheet of polyvinyl alcohol is heated and stretched from three to eight times its original length. By this operation the molecules are oriented in the direction of stress and when the sheet is impregnated with iodine it exhibits strong dichroism. This polaroid is colourless and transmits more light than herapathite polaroid.

Land and Rogers later discovered that when the stretched polyvinyl alcohol film is heated with a catalyst like hydrochloric acid it slightly darkens but exhibits strong dichroism. As the film does not contain any dyestuff, it is very stable and is not bleached by strong sunlight. It is called ***K*-polaroid** and is extensively employed in automobile head lights and wind screen.

Polaroids have wide applications in everyday life e.g., in sun glasses, automobile headlights, wind screen, windows of railway trains, aeroplanes, in stereoscopic motion pictures etc.

13.11 HUYGENS' THEORY OF DOUBLE REFRACTION AND THE NATURE OF WAVE SURFACES IN UNIAXIAL CRYSTAL :

Huygens with the help of his theory of secondary wavelets gave a suitable explanation of the phenomenon of double refraction in a uniaxial crystal. According to him every point of a doubly refracting crystal disturbed by the incident light becomes the source of two secondary wavelets— spherical for *O*-ray and ellipsoid of revolution about the optic axis for the *E*-ray. The two wave surfaces touch each other along the optic axis through the point of origin of the wavelets. This theory can account for the following facts regarding double refraction :

(i) There are two refracted rays— *O*- and *E*-rays.

(ii) *O*-ray travels with equal velocity in all directions and obeys the ordinary laws of refraction.

(iii) *E*-ray does not obey the laws of refraction.

(iv) Optical properties of uniaxial crystals are perfectly symmetrical about the optic axis.

(v) Along the optic axis *O*- and *E*-rays travel with equal velocity and there is no double refraction.

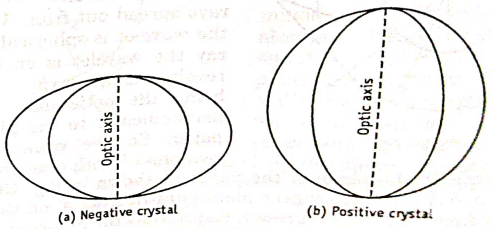


Fig. 13.11-1

The nature of the secondary wave surfaces is shown in Fig. 13.11-1.

Crystals like calcite in which *O*-ray travels with smaller velocity than *E*-ray (i.e., $n_e < n_o$) in the direction normal to the optic axis are called *negative crystals*. In this case the spherical wave surface is considered within the ellipsoid of revolution. Crystals like quartz in which *O*-ray travels with greater velocity than *E*-ray (i.e., $n_e > n_o$) in the direction normal to the optic axis are called *positive crystals* and here the ellipsoid of revolution is considered within the sphere.

13.12 HUYGENS' CONSTRUCTION OF WAVE SURFACES IN A UNIAXIAL CRYSTAL :

For construction of wave surfaces on the plane of incidence we consider three distinct cases :

Case I : Optic axis perpendicular to the plane of incidence and parallel to the crystal surface.

Let AB be the trace of plane wavefront at any instant incident obliquely on the surface of a uniaxial negative crystal, such as calcite, whose axis is on the surface and perpendicular to the plane of incidence. The section of the plane wavefront and that of the surface of calcite (both of which are supposed to be perpendicular to the plane of paper) are repersented respectively by AB and LAA' in the Fig. 13.12-1. Here any line through the crystal drawn perpendicular to the plane of the paper is an optic axis. At the instant when the incident wavefront passes through A, the secondary wavelets start from A and spread within the crystal with different velocities. During the time the edge B moves to A', the secondary wavelets for E- and O-rays spread out from A. For O-ray the wavelet is spherical and for E-ray the wavelet is an ellipsoid of revolution, the axis of revolution being the optic axis, here it is perpendicular to the plane of the paper. So the equatorial cross-sections of both the wavelets will be a semicircle in the plane of the paper as shown in Fig. 13.12-1. The traces $A'C$ and $A'C'$of the tangent planes drawn from A' on the ordinary and extraordinary wavelets represent respectively the refracted wavefronts for O- and E-waves after a time $t = BA'/v$, where v is the velocity of light in the upper medium. Hence AC and AC' are respectively the refracted rays for ordinary and extraordinary light in the crystal and they should lie in the plane of incidence.

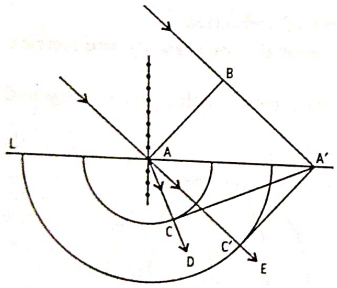


Fig.13.12-1

Note that here,

$$\frac{BA'}{v} = \frac{AC}{v_o} = \frac{AC'}{v_e}$$

where v_o and v_e are respectively the velocities of O- and E-rays in the crystal.

Case II : Optic axis parallel to the refracting face of the crystal and situated in the plane of incidence.

Let AB be the trace of a plane wavefront incident obliquely on the plane refracting surface LA' of a negative crystal. Both the wavefront and the refracting surface are perpendicular to the plane of the paper. Here any line drawn parallel to LA' in the plane of the paper represents an optic axis. In the time edge B moves to A' the wavelets for E- and O-rays from A will move to the positions as shown in Fig. 13.12-2, where,

$$\frac{BA'}{v} = \frac{AN}{v_o} = \frac{AN'}{v_e}$$

in which v is the velocity of light in the upper medium, v_o and v_e are respectively the velocities of O- and E-rays in the crystal in a direction normal to the optic axis. Thus after a time $t\,(= BA'/v)$ the section of the ordinary wavelet by the incident plane will be a semi-circle while that of the extraordinary wavelet by the incident plane will be a semi-ellipse both touching along the minor axis of the ellipse which is also the axis of the crystal. $A'C$ and $A'C'$ are the traces of tangent planes drawn from A' on the ordinary and extraordinary wavelets and are, therefore, representing respectively the refracted wavefronts for ordinary and extraordinary light after a time t. Hence AC and $AC'$are respectively the refracted ordinary and extraordinary rays in the crystal.

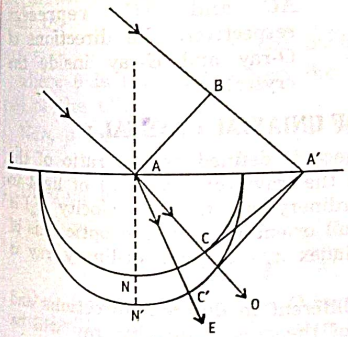


Fig.13.12-2

Case III : Optic axis lies on the plane of incidence but perpendicular to the refracting surface.

Let AB be the plane wavefront, incident obliquely on the surface of a uniaxial negative crystal whose axis is perpendicular to the surface but lies in the incident plane (Fig.13.12-3). As the secondary waves of both O-ray and E-ray travel with equal velocity along the optic axis AO of the crystal, the O-wave surface SOS' will touch the E-wave surface EOE' along the optic axis AO.

During the time t ($t = BA'/v$, where v is the velocity of light in the upper medium) the secondary wave from B reaches A', the ordinary and extraordinary secondary waves from A will move to the positions shown in Fig. 13.12-3 where,

$$t = \frac{BA'}{v} = \frac{AS}{v_o} = \frac{AE}{v_e}$$

The sections of the wavelets by the plane of incidence are semi-elliptical (EOE') and semi-circular (SOS') for E- and O-rays respectively. The tangents $A'C$ and $A'C'$ drawn from A' on the secondary waves of O-ray and E-ray respectively will represent the sections of the ordinary and extraordinary wavefronts within the crystal after time t. The straight lines AC and AC' represent respectively the directions of O-ray and E-ray inside the crystal.

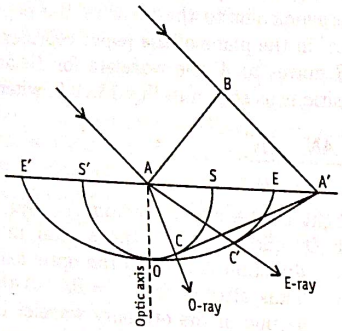


Fig. 13.12-3

13.13 REFRACTIVE INDICES OF UNIAXIAL CRYSTAL :

Refractive index (n) of a substance is defined as the ratio of the velocity of light in vacuum ($= c$) to the wave-velocity ($= v$) of light in that substance. Thus $n = c/v$. For ordinary ray, its wave-velocity (v_o) is the same in all directions and for all orientations of the optic axis of the crystal. Hence the refractive index $n_o = c/v_o$ for ordinary ray is constant for the crystal.

For E-ray, the wave-velocity is different in different directions and consequently the refractive index of the crystal for this ray will be different in different directions. If v_e is the wave-velocity of E-ray along the direction perpendicular to the optic axis then refractive index for the E-ray along this direction is $n_e = c/v_e$. This is known as *principal refractive index* for E-rays. Along the optic axis both the E- and O-rays travel with same velocity and hence will have the same refractive index in this direction.

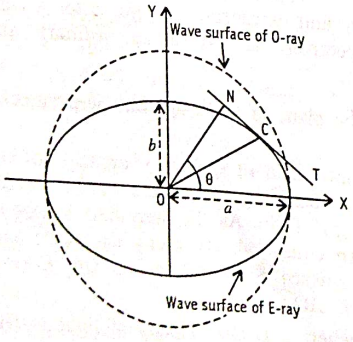


Fig. 13.13-1

To define the refractive index for E-rays at any direction through the crystal we are to distinguish between *ray-velocity* and *wave-velocity*. Fig. 13.13-1 shows that the direction OC of E-ray

and the normal ON to the E-ray wavefront TN do not coincide except along and perpendicular to the optic axis. The rate of increase of the line joining the origin (O) of a secondary wave and the point of contact C of the wavefront with that secondary wave is known as the *ray-velocity* of the wave. The rate of increase of the normal (ON) to the wavefront represents the *wave-velocity* of the wave. The wave-velocity of E-ray differs from its ray-velocity but they become equal along and perpendicular to the optic axis. For any direction making an gle θ with the optic axis the refractive index for E-ray is given by,

$$n_\theta = \frac{\text{velocity of light in vacuum}(c)}{\text{wave velocity for } E\text{ - rays in this direction}}$$

Now from coordinate geometry the length (p) of the perpendicular ON drawn from the centre (O) of the ellipse on the tangent TN is given by,

$$p^2 = a^2 \cos^2\theta + b^2 \sin^2\theta \qquad \ldots(13.13\text{-}1)$$

where θ is the angle which the normal ON makes with the major axis of the ellipse.

Now, $p = v_\theta t$, $a = v_o t$, $b = v_e t$. Hence from Eq. (13.13-1) we get,

$$v_\theta^2 = v_o^2 \cos^2\theta + v_e^2 \sin^2\theta$$

Dividing by c^2 we get,

$$\frac{1}{n_\theta^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2} \qquad \ldots(13.13\text{-}2)$$

Thus knowing n_o (r.i. for O-ray), n_e (principal r.i. for E-ray), the refractive innedx n_θ for E-rays in any direction can be calculated.

The quantity $\Delta n = n_e - n_o$ is often referred to as the *birefringence*. A uniaxial crystal is denoted as either $+ve$ or $-ve$ depending on the sign of Δn.

Measurement of principal refractive indices :

To measure the principal refractive indices a prism is cut from the crystal with the optic axis parallel to the edges of the prism. The pirsm is placed on a table of a spectrometer. When a parallel beam of monochromatic light is made incident on the prism, the plane of incidence becomes perpendicular to the axis of the crystal. The emergent beam consists of two beams— one for E-ray and another for O-ray (Fig. 13.13-2).

By measuring the angle (A) of the prism and minimum deviations (D_m) for O-ray and E-ray we can find the refractive indices of the crystal for O-ray and principal refractive index for E-ray in a direction at right